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Super-radiance near conducting and plasma surfaces

M Babiker
Physics Department, University of Khartoum, Khartoum, Sudan

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Abstract. In free space, the presence of an atom in its ground state near an excited atom undergoing an electric dipole transition, leads to a modification of the transition rate. This is shown to be modified further if the system of two atoms is located near a macroscopic surface. For small inter-atomic separations, $R \ll ƛ_0$, ($ƛ_0$ is the reduced electric dipole wavelength) and for distances z, $z \ll ƛ_0/2$ of the system from a perfectly conducting surface, the transition rate for the symmetric state, when both atomic dipoles are oriented perpendicular to the surface, is *four* times the rate for an isolated single atom. The antisymmetric state is stable. For a configuration in which dipoles are oriented parallel to the surface, both the symmetric and the antisymmetric states are stable. When the behaviour of the same system is investigated near the surface of a semi-infinite electron plasma, serving as a model for a metal, it is found that, in addition to the behaviour in the perfect conductor case, contributions from surface plasmon modes, and other contributions accounting for the finiteness of the plasma frequency are also exhibited.

1. Introduction

It is well known (see for instance Power 1967), that in a region of space free of boundaries, an excited atom undergoing an electric dipole transition, can decay at twice the normal rate, when a second atom in its ground state is also present in its neighbourhood. The decay rate, in general, depends on the relative orientation of the dipole moments of, and is a function of the distance between, the two atoms. This effect was first pointed out by Dicke (1954). A single atom near a conducting surface is also known to exhibit such an effect. In this case it can be explained as being due to a cooperative emission of a correlated state of the atom and its image in the conducting surface (Milonni and Knight 1973).

If in addition to the presence of the conducting surface, another atom in its ground state is also present within a short distance from the original atom, the decay rate will be modified even further, depending on the relative orientation of the atomic dipoles. The prime motivation of this paper is the investigation of this problem.

In a region of space far removed from the conducting surface, the two-atom system exhibits radiation effects of the type discussed by Dicke (1954). Close to the conducting surface, it is found that the transition rate is particularly enhanced when both the atomic dipole moments are oriented perpendicular to the surface. Very close to the conducting surface, the symmetric state decays at four times the decay rate of a single isolated atom. The presence of the conductor effectively multiplies the already enhanced decay rate by another factor of two.

The perfectly conducting surface is most widely discussed in problems involving the effects of macroscopic bodies on atomic properties. In the recent literature, the model was employed by Barton (1970, 1974); Babiker and Barton (1972); Milonni and Knight (1973); and Philpott (1973). The simplicity of such a model when employed in actual calculations, stems from the simple boundary conditions imposed by Maxwell's equations at the surface of a perfect conductor, which is taken as impermeable to electromagnetic fields at all frequencies, however high. In practice, however, field penetration across the boundaries of material media is a physical reality, and for most practical cases the perfect conductor model is not adequate.

A model that illustrates the effects of field penetration is the metallic electron plasma, which has also been discussed in the recent literature (Elson and Ritchie 1971). Although the boundary conditions, and consequently the mode expansion of the electromagnetic fields, are quite involved here, the model permits the investigation of surface effects of a novel type. In addition to the usual photon modes, there exist other modes, called the surface plasmons, which can have important physical effects on atomic properties near a metallic surface. Another characteristic of the plasma model is that, in the limit of high plasma frequencies, the results for the perfect conductor case emerge. Quantum mechanical treatments using this model have already been reported. For a single atom, the frequency shifts have been discussed by Babiker and Barton (1976) for electric dipole transitions, and by Babiker (1975) for hyperfine structure transitions, while decay rates have been discussed by Philpott (1975).

In this paper, we shall be concerned with the emission effects mentioned earlier, for a system of two atoms in the vicinity of a metallic surface. We elaborate first on the perfect conductor case and briefly outline how the procedure and results are modified for the case of the plasma model.

The decay rate of the system can, in principle, be calculated for a general atomic dipole orientation, but, for illustration purposes, we shall confine our attention to a few special configurations for which the calculations are relatively manageable and the results easy to interpret. The configurations discussed here are so chosen that the decay rates into all mode channels are taken into account. For the case of the plasma, we shall confine ourselves to only one of the configurations discussed in the perfect conductor case.

In § 2 we define the quantized transverse vector potential in the presence of an infinite conducting surface and explain how this leads to the expression of the decay rate of a single excited atom situated at a fixed distance from the surface. The results are needed for meaningful comparison with the results for the two-atom system discussed in the later sections. In § 3 we consider the two-atom system in front of a perfect conductor and give fairly detailed accounts for some special cases for which the dipole orientations are simple. In § 4 we consider the plasma surface in place of the conducting surface and illustrate how the calculations for one of the configurations proceed. Section 5 contains a summary and some comments.

2. Single atom near conductor

2.1. Quantized radiation field

We shall assume that the plane $z = 0$ defines the surface of the metal, which is taken to occupy the half space, $z < 0$. Atomic systems will be on the vacuum side, $z > 0$, at distances which always exceed the average atomic size.

For a perfectly conducting metal, the vector potential may be quantized in the Coulomb gauge, div $\boldsymbol{A} = 0$. An appropriate mode expansion, satisfying the boundary conditions at $z=0$, is given by (Barton 1974)

$$A(x)=\int_{-\infty}^{\infty} dk\int_0^{\infty} dq\left(\frac{1}{\pi^2\omega}\right)^{1/2}\left[a_s(k,q)\hat{k}\times\hat{z}\sin qz+a_p(k,q)\right.$$
$$\left.\times\left(\hat{k}\frac{iq}{\omega}\sin qz-\hat{z}\frac{k}{\omega}\cos qz\right)\right]e^{ik.\rho}+(\text{Hermitean conjugate}), \tag{2.1}$$

where $\omega^2=k^2+q^2$, ρ is the coordinate parallel to the surface, unit vectors are designated by carets and the $a_s(k,q)$ and $a_p(k,q)$ are the usual annihilation operators for the s photons and the p photons, respectively,

$$[a_r(k,q),a^{\dagger}_{r'}(k',q')]=\delta_{rr'}\delta(k-k')\delta(q-q'); \qquad (r,r'=s,p). \tag{2.2}$$

2.2. Transition rate for a single atom

Consider a single neutral atom in the electric dipole approximation, with its nucleus fixed at a distance z from the conducting surface. We shall use the Hamiltonian of Power and Zienau (1959) and write

$$H=H_0+H_{\text{int}}, \tag{2.3}$$

where H_0 denotes the Hamiltonian for the radiation field and the unperturbed atom, while H_{int} is given by

$$H_{\text{int}}=-d.E=d.\dot{A}(x). \tag{2.4}$$

In equation (2.4) x is the coordinate of the nucleus, d is the electric dipole moment operator of the atom and we use natural units $\hbar=c=1$.

The transition rate of an excited state $|i\rangle$ to a lower state $|f\rangle$ is, to lowest order, given by

$$\gamma(z)=2\pi\sum_{r=s,p}\int_{-\infty}^{\infty} dk\int_0^{\infty} dq|\langle i;0|H_{\text{int}}|f;k,q,r\rangle|^2\delta(E_i-E_f-\omega(k,q,r)). \tag{2.5}$$

The total decay rate is therefore given by the sum of the separate decay rates into the s and p photon channels

$$\gamma=\gamma_s+\gamma_p. \tag{2.6}$$

We shall consider the cases when the atomic dipole moment is oriented either parallel or perpendicular to the surface. When d is perpendicular to the surface, the contribution to $\gamma^{\perp}$ comes only from the p photons

$$\gamma_s^{\perp}=0, \qquad \gamma^{\perp}=\gamma_p^{\perp}. \tag{2.7}$$

The evaluation of $\gamma_p^{\perp}$ is straightforward: substitute from (2.4) into (2.5) and use (2.1) and (2.2). The resulting integrals over k and q can be performed to obtain the closed form

$$\gamma^{\perp}(z)=\gamma_0(1+3F(2\omega_0 z)), \tag{2.8}$$

where $\omega_0 = E_i - E_f$,

$$F(2\omega_0 z) = \frac{\sin 2\omega_0 z}{(2\omega_0 z)^3} - \frac{\cos 2\omega_0 z}{(2\omega_0 z)^3} \equiv \tfrac{1}{3}(J_2(2\omega_0 z) + J_0(2\omega_0 z)), \tag{2.9}$$

and

$$\gamma_0 = \tfrac{4}{3}|\langle i|d|f\rangle|^2 \omega_0^3 \equiv \tfrac{4}{3} d^2 \omega_0^3, \tag{2.10}$$

is the transition rate for the isolated atom. In equation (2.9) J_0 and J_2 are Bessel functions of the zeroth and second order, respectively.

When the dipole is oriented parallel to the surface, there are contributions to the decay rate from both the s and the p photons and we have

$$\gamma^{\parallel} = \gamma_s^{\parallel} + \gamma_p^{\parallel}. \tag{2.11}$$

In this case one finds the expressions

$$\gamma_s^{\parallel}(z) = \frac{3}{4}\gamma_0\left(1 - \frac{\sin 2\omega_0 z}{2\omega_0 z}\right), \tag{2.12}$$

$$\gamma_p^{\parallel}(z) = \frac{1}{4}\gamma_0\left[1 + 3\left(2F(2\omega_0 z) - \frac{\sin 2\omega_0 z}{2\omega_0 z}\right)\right]. \tag{2.13}$$

Hence

$$\gamma^{\parallel}(z) = \gamma_0(1 + \tfrac{3}{2}G(2\omega_0 z)). \tag{2.14}$$

where

$$G(2\omega_0 z) = F(2\omega_0 z) - \frac{\sin 2\omega_0 z}{2\omega_0 z} \equiv \tfrac{1}{3}(J_2(2\omega_0 z) - 2J_0(2\omega_0 z)). \tag{2.15}$$

Equations (2.8) and (2.14) give the modified decay rates due to the presence of the conducting surface. The second terms in these expressions are image corrections which are particularly important when the atom is close to the surface. For small distances z, satisfying $2\omega_0 z \ll 1$, we have

$$F(2\omega_0 z) \simeq \tfrac{1}{3}; \qquad G(2\omega_0 z) \simeq -\tfrac{2}{3}. \tag{2.16}$$

The inequality $2\omega_0 z \ll 1$ is equivalent to the statement that the distance of the atom from the surface is less than half the reduced dipole wavelength ($z \ll \lambda\!\!\!^{-}_0/2$). Under these conditions

$$\gamma^{\perp} \simeq 2\gamma_0; \qquad \gamma^{\parallel} \simeq 0. \tag{2.17}$$

These results show that, close to the surface, the excited state of an atom whose dipole is oriented perpendicular to the surface decays at twice the rate of an isolated atom (super-radiance). When the dipole is oriented parallel to the surface, the excited state is stable (sub-radiance).

3. Two-atom system near conductor

Consider a system of two identical atoms, labelled 1 and 2, of which one is excited and the other is in its ground state, with their nuclei positioned at fixed distances from the perfectly conducting surface described in § 2. In this case we write

$$H = H_0 + H_{\rm int}, \tag{3.1}$$

where H_0 now designates the Hamiltonian of the unperturbed two-atom system plus the radiation field, while H_{int} is given in the electric dipole approximation by

$$H_{\text{int}} = \sum_{i=1,2} \boldsymbol{d}_i \cdot \dot{\boldsymbol{A}}(\boldsymbol{x}_i). \tag{3.2}$$

The quantized vector potential $\boldsymbol{A}$ is still given by equation (2.1) and $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are the coordinates of the nuclei.

Both atoms essentially see the same amplitude of the radiation field, and therefore dipole selection rules apply to the decay of the combined system. The initial state is either the symmetric (+) state or the antisymmetric (−) state

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|\mathrm{i}\rangle_1|\mathrm{f}\rangle_2 \pm |\mathrm{i}\rangle_2|\mathrm{f}\rangle_1), \tag{3.3}$$

for which the final state is always

$$|\psi_0\rangle = |\mathrm{f}\rangle_1|\mathrm{f}\rangle_2. \tag{3.4}$$

We calculate the decay rates $\Gamma^{(\pm)}$ for both types of states defined by equation (3.3). The golden rule gives

$$\Gamma^{(\pm)} = 2\pi \sum_{r=s,p} \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{k} \int_0^{\infty} \mathrm{d}q |\langle\psi_\pm; 0|H_{\text{int}}|\psi_0; \boldsymbol{k}, q, r\rangle|^2 \delta(E_\mathrm{i} - E_\mathrm{f} - \omega(\boldsymbol{k}, q, r)). \tag{3.5}$$

In the most general case, $\Gamma^{(\pm)}$ are functions of the atomic positions as well as the relative orientation of the atomic dipole moment vectors. In what follows, we shall specialize in some simple configurations and discuss the decay rates for each of these configurations.

3.1. *Configuration (a)*

Here the atoms are situated at equal distances $z_1 = z = z_2$, from the surface, with their dipoles both pointing in the $\hat{z}$ direction. We choose axes as shown in figure 1, with the atoms fixed at the space points $(0, 0, z)$ and $(R, 0, z)$.

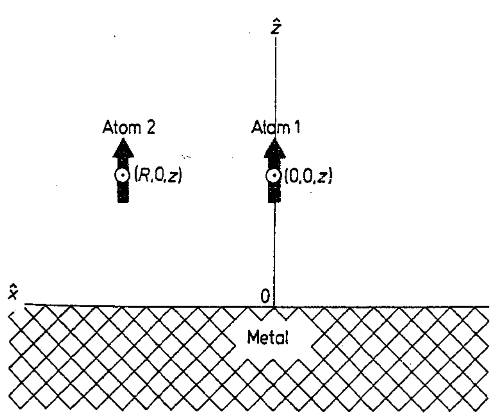

Figure 1. Configuration (*a*).

Using equations (3.2), (2.2), and (2.1) we obtain from equation (3.5)

$$\Gamma^{(\pm)} = 2\pi \int_0^\infty k\, dk \int_0^{2\pi} d\phi \int_0^\infty dq \Big(\frac{\omega}{2\pi^2} \frac{k^2}{\omega^2} \cos^2 qz [d_1^2 + d_2^2 + d_1 d_2 (e^{ikR\cos\phi} + e^{-ikR\cos\phi})] \delta(\omega_0 - \omega) \Big). \tag{3.6}$$

Since the atoms are identical, we can put $d_1 = d = d_2$, and define the functions $S^{(\pm)}(kR)$:

$$S^{(\pm)}(kR) = \frac{1}{4\pi d^2} \int_0^{2\pi} d\phi [d_1^2 + d_2^2 \pm d_1 d_2 (e^{ikR\cos\phi} + e^{-ikR\cos\phi})] = 1 \pm J_0(kR), \tag{3.7}$$

where J_0 is the Bessel function of the zeroth order. Equation (3.6) then becomes

$$\Gamma^{(\pm)} = 4\pi d^2 \frac{2\pi}{2\pi^2} \int_0^\infty k\, dk \int_0^\infty dq \frac{k^2}{\omega} \cos^2 qz S^{(\pm)}(kR) \delta(\omega_0 - \omega). \tag{3.8}$$

We introduce new variables

$$\omega = (k^2 + q^2)^{1/2}, \qquad \mu = q/\omega, \tag{3.9}$$

for which the Jacobian is $\omega(1-\mu^2)^{-1/2}$, and the limits of the integration are as shown below. Equation (3.8) becomes

$$\Gamma^{(\pm)} = 4d^2 \int_0^\infty d\omega \int_0^1 d\mu \omega^3 (1-\mu^2) \cos^2 \mu\omega z S^{(\pm)}(\omega, \mu, R) \delta(\omega_0 - \omega). \tag{3.10'}$$

The ω integration immediately gives

$$\Gamma^{(\pm)} = 4d^2 \omega_0^3 \int_0^1 d\mu (1-\mu^2) \cos^2 \mu\omega_0 z S^{(\pm)}(\omega_0, \mu, R), \tag{3.10}$$

where

$$S^{(\pm)}(\omega_0, \mu, R) = 1 \pm J_0(\omega_0 (1-\mu^2)^{1/2} R). \tag{3.11}$$

The full expressions given by equation (3.10) define the decay rates of the (+) and (−) states for any z and R. These complicated expressions, however, simplify in some special but interesting limits. We define the 'small-R' limit by the inequality $\omega_0 R \ll 1$ for which the inter-atomic distance is less than the reduced dipole wavelength, $\lambda\!\!\!^-_0$. Similarly we define the 'small-z' limit by the inequality $2\omega_0 z \ll 1$. In the latter limit, z is still required to exceed the average atomic size. We also note that these limits are interchangeable and mutually compatible, so that they can both be applied simultaneously. In the limit 'small R' and 'small z' the atoms are separated from each other and from the surface by distances less than the reduced wavelength $\lambda\!\!\!^-_0$ and half the reduced wavelength $\lambda\!\!\!^-_0/2$ respectively. The symbols $\doteq$, $\overset{\cdot}{=}$ and $\div$ will be used to designate equalities under the limits 'small R', 'small z' and 'small R and small z', respectively.

Prior to applying the above mentioned limits to the decay rates given by equation (3.10), we first note that this expression can be separated into z-independent and z-dependent parts. We write

$$\Gamma^{(\pm)} = \Gamma_a^{(\pm)}(R) + \Gamma_b^{(\pm)}(R, z), \tag{3.12}$$

where

$$\Gamma_a^{(\pm)}(R)=\tfrac{3}{2}\gamma_0\int_0^1 d\mu(1-\mu^2)S^{(\pm)}(\omega_0,\mu,R), \tag{3.13}$$

$$\Gamma_b^{(\pm)}(R,z)=\tfrac{3}{2}\gamma_0\int_0^1 d\mu(1-\mu^2)\cos 2\mu\omega_0 z S^{(\pm)}(\omega_0,\mu,R). \tag{3.14}$$

$\Gamma_a^{(\pm)}$ are completely independent of the presence of the conducting surface and are functions of R only. Accordingly we identify $\Gamma_a^{(\pm)}$ as the decay rates for the isolated two-atom system. In the 'small-R' limit we have

$$J_0(\omega_0(1-\mu^2)^{1/2}R)\doteq 1+O(\omega_0^2R^2). \tag{3.15}$$

Hence equation (3.13) gives

$$\Gamma_a^{(\pm)}\doteq\tfrac{3}{2}\gamma_0\int_0^1 d\mu(1-\mu^2)[1\pm(1+O(\omega_0^2R^2))]. \tag{3.16}$$

So, for the symmetric (+) state

$$\Gamma_a^{(+)}\doteq 2\gamma_0, \tag{3.17}$$

while for the antisymmetric (−) state

$$\Gamma_a^{(-)}\doteq 0+O(\omega_0^5R^2). \tag{3.18}$$

These are the Dicke results for an isolated system of two atoms.

The z-dependent parts, $\Gamma_b^{(\pm)}$, are defined by equation (3.14). In interpreting these results, it is best to impose the 'small-R' limit first and reduce the expression for a general z. Thus

$$\Gamma_b^{(\pm)}\doteq\tfrac{3}{2}\gamma_0\int_0^1 d\mu(1-\mu^2)\cos 2\mu\omega_0 z[1\pm(1+O(\omega_0^2R^2))]. \tag{3.19}$$

This gives for the (+) state

$$\Gamma_b^{(+)}\doteq 6\gamma_0 F(2\omega_0 z), \tag{3.20}$$

and for the (−) state,

$$\Gamma_b^{(-)}\doteq 0+O(\omega_0^5R^2). \tag{3.21}$$

If, further, we impose the 'small-z' limit, we obtain

$$\Gamma_b^{(+)}\doteq 2\gamma_0. \tag{3.22}$$

It follows from equations (3.17), (3.18), (3.21) and (3.22) that for 'small R and small z' we have

$$\Gamma^{(+)}\doteq 4\gamma_0, \tag{3.23}$$

$$\Gamma^{(-)}\doteq 0+O(\omega_0^5R^2). \tag{3.24}$$

The above results show that, close to the surface ($z\ll\lambda_0/2$) the symmetric state of a system of an excited atom and another atom in its ground state, separated by a distance less than λ_0, will radiate at four times the decay rate of a completely isolated system of one atom. The antisymmetric state of the system is stable.

Finally we note that for 'small z' and a general R, equations (3.13) and (3.14) combine to give

$$\Gamma^{(\pm)} = 3\gamma_0 \int_0^1 \mathrm{d}\mu (1-\mu^2) S^{(\pm)}(\omega_0, \mu, R), \tag{3.25}$$

from which we infer that, close to the surface, the isolated decay rates of the system are effectively multiplied by a factor of two. For large R and large z the decay rates reduce to the decay rates for single isolated atoms, as they should. This can be deduced directly from equations (3.13) and (3.14) under these conditions.

3.2. *Configuration (b)*

The second configuration is schematically shown in figure 2, in which the atoms are situated on the z axis with their dipoles still pointing in the $\hat{z}$ direction. In this case one obtains for the decay rates

$$\Gamma^{(\pm)} = d^2 \frac{2\pi}{2\pi^2} \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{k} \int_0^{\infty} \mathrm{d}q \frac{k^2}{\omega} (\cos qz_1 \pm \cos qz_2)^2 \delta(\omega_0 - \omega). \tag{3.26}$$

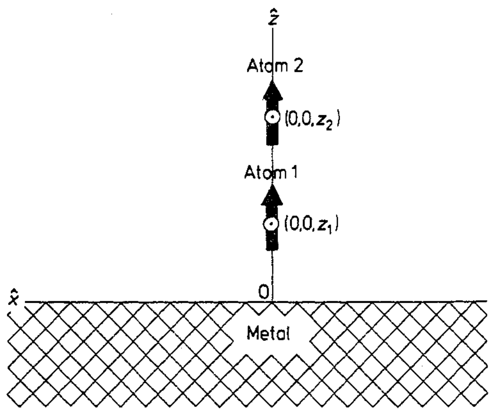

Figure 2. Configuration (*b*).

Define R and Z by

$$R = z_2 - z_1, \qquad Z = (z_1 + z_2)/2. \tag{3.27}$$

Elementary trigonometry will then reduce equation (3.26) to the following form:

$$\Gamma^{(\pm)} = 2d^2 \int_0^{\infty} k\mathrm{d}k \int_0^{\infty} \mathrm{d}q \frac{k^2}{\omega} (1 \pm \cos 2qZ)(1 \pm \cos qR) \delta(\omega_0 - \omega), \tag{3.28}$$

which becomes, on transforming to the variables ω and μ, defined by equation (3.9), and performing the ω integration:

$$\Gamma^{(\pm)} = \tfrac{3}{2}\gamma_0 \int_0^1 \mathrm{d}\mu (1-\mu^2)(1 \pm \cos 2\mu\omega_0 Z \pm \cos \mu\omega_0 R + \cos \mu\omega_0 R \cos 2\mu\omega_0 Z). \tag{3.29}$$

Again we note that the decay rates separate into z-independent and z-dependent parts

$$\Gamma^{(\pm)}=\Gamma_{\rm a}^{(\pm)}(R)+\Gamma_{\rm b}^{(\pm)}(z,R). \tag{3.30}$$

$\Gamma_{\rm a}^{(\pm)}$ correspond to the expressions for the isolated two-atom system and $\Gamma_{\rm b}^{(\pm)}$ are the corresponding corrections due to the presence of the surface. In this case, we obtain for $\Gamma_{\rm a}^{(\pm)}$ the exact result

$$\Gamma_{\rm a}^{(\pm)}=\tfrac{3}{2}\gamma_0\int_0^1 {\rm d}\mu(1-\mu^2)(1\pm\cos\mu\omega_0R)=\gamma_0(1\pm 3F(\omega_0R)). \tag{3.31}$$

In the 'small-R' limit, the Dicke results are obtained,

$$\Gamma_{\rm a}^{(+)}\doteq 2\gamma_0, \qquad \Gamma_{\rm a}^{(-)}\doteq 0+{\rm O}(\omega_0^5R^2). \tag{3.32}$$

The z-dependent parts, $\Gamma_{\rm b}^{(\pm)}$ are given by

$$\Gamma_{\rm b}^{(\pm)}=\tfrac{3}{2}\gamma_0\int_0^1 {\rm d}\mu(1-\mu^2)\cos 2\mu\omega_0 Z(\cos\mu\omega_0R\pm 1). \tag{3.33}$$

In the 'small-R' limit we obtain

$$\Gamma_{\rm a}^{(\pm)}\doteq\tfrac{3}{2}\gamma_0\int_0^1 {\rm d}\mu(1-\mu^2)\cos 2\mu\omega_0 Z[\pm 1+(1+{\rm O}(\omega_0^2R^2))], \tag{3.34}$$

which yields for the $(-)$ state

$$\Gamma_{\rm b}^{(-)}\doteq 0+{\rm O}(\omega_0^5R^2), \tag{3.35}$$

while for the $(+)$ state, we get

$$\Gamma_{\rm b}^{(+)}\doteq 6\gamma_0F(2\omega_0Z). \tag{3.36}$$

If, further, we impose the 'small-Z' limit to the above results we obtain

$$\Gamma^{(+)}\doteq 4\gamma_0, \qquad \Gamma^{(-)}\doteq 0+{\rm O}(\omega_0^5R^2). \tag{3.37}$$

The behaviour mentioned in the previous case is again concluded.

3.3. *Configuration* (c)

Here the two atoms are positioned on the z axis at $(0,0,z_1)$ and $(0,0,z_2)$ with their dipoles pointing in the $\hat{x}$ direction. This is schematically shown in figure 3.

In contrast to the previous two cases, where the decay is only into the p-photon channel, in the present configuration both the s-photon and the p-photon channels are involved,

$$\Gamma^{(\pm)}=\Gamma_s^{(\pm)}+\Gamma_p^{(\pm)}. \tag{3.38}$$

The method of calculation is essentially the same as described before. We find

$$\Gamma_s^{(\pm)}=\tfrac{3}{4}\gamma_0\int_0^1 {\rm d}\mu[1-\cos\mu\omega_0R\cos 2\mu\omega_0Z\pm(\cos\mu\omega_0R-\cos 2\mu\omega_0Z)], \tag{3.39}$$

$$\Gamma_p^{(\pm)}=\tfrac{3}{4}\gamma_0\int_0^1 {\rm d}\mu\,\mu^2[1-\cos\mu\omega_0R\cos 2\mu\omega_0Z\pm(\cos\mu\omega_0R-\cos 2\mu\omega_0Z)]. \tag{3.40}$$

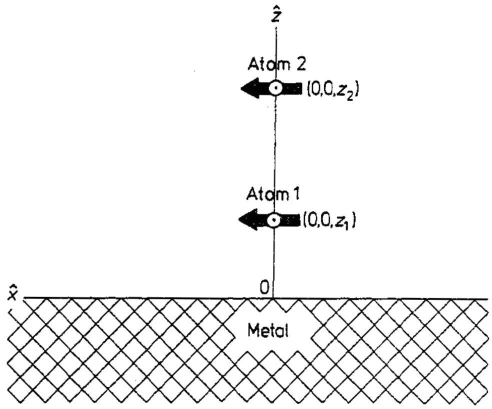

Figure 3. Configuration (*c*).

Accordingly, the total decay rates are given by

$$\Gamma^{(\pm)}=\tfrac{3}{4}\gamma_0\int_0^1 \mathrm{d}\mu(1+\mu^2)[1-\cos\mu\omega_0 R\cos 2\mu\omega_0 Z\pm(\cos\mu\omega_0 R-\cos 2\mu\omega_0 Z)] \quad (3.41)$$

which separates into z-independent and z-dependent parts:

$$\Gamma^{(\pm)}=\Gamma_{\mathrm{a}}^{(\pm)}(R)+\Gamma_{\mathrm{b}}^{(\pm)}(Z,R). \quad (3.42)$$

The z-independent parts are

$$\Gamma_{\mathrm{a}}^{(\pm)}(R)=\tfrac{3}{4}\gamma_0\int_0^1 \mathrm{d}\mu(1+\mu^2)(1\pm\cos\mu\omega_0 R)=\gamma_0\mp\tfrac{3}{2}\gamma_0 G(\omega_0 R), \quad (3.43)$$

which reduce to the familiar results

$$\Gamma_{\mathrm{a}}^{(+)}\doteq 2\gamma_0, \qquad \Gamma_{\mathrm{a}}^{(-)}\doteq 0+\mathrm{O}(\omega_0^5 R^2), \quad (3.44)$$

in the 'small-R' limit.

The z-dependent parts are given by

$$\Gamma_{\mathrm{b}}^{(\pm)}(R,Z)=-\tfrac{3}{4}\gamma_0\int_0^1 \mathrm{d}\mu(1+\mu^2)(\cos\mu\omega_0 R\pm 1)\cos 2\mu\omega_0 Z, \quad (3.45)$$

from which we obtain in the 'small-R' limit

$$\Gamma_{\mathrm{b}}^{(+)}\doteq 3\gamma_0 G(2\omega_0 Z); \qquad \Gamma_{\mathrm{b}}^{(-)}\doteq 0+\mathrm{O}(\omega_0^5 R^2). \quad (3.46)$$

Finally we impose the 'small-Z' limit. We get from equation (3.46)

$$\Gamma_{\mathrm{b}}^{(+)}\doteq -2\gamma_0; \qquad \Gamma_{\mathrm{b}}^{(-)}\doteq 0+\mathrm{O}(\omega_0^5 R^2). \quad (3.47)$$

The total decay rates in the 'small-R and small-Z' limit are obtained by adding contributions from equations (3.44) and (3.47). The result can be written as

$$\Gamma^{(\pm)}\doteq 0+\mathrm{O}(\omega_0^5 R^2)+\mathrm{O}(\omega_0^5 Z^2). \quad (3.48)$$

We conclude that, under the above conditions, and when the atomic dipoles are oriented parallel to the surface, both the (+) and the (−) states are stable. In contrast to the behaviour encountered in configurations (*a*) and (*b*), where the presence of the surface leads to super-radiance of the (+) states and sub-radiance of the (−) states, the presence of the surface leads to sub-radiance of both types of states in configuration (*c*). For a general orientation the behaviour of the system near the surface is obviously more complicated, but for both types of states an intermediate behaviour between sub-radiance and super-radiance is to be expected.

4. Atoms near a plasma surface

The decay rate for a single atom near the surface of an electron plasma occupying the half space $z<0$, has been considered by Philpott (1975). His treatment, however, excludes the photon modes transmitted into the plasma and is applicable only for frequency regions for which the effective dielectric function of the plasma is negative. The transverse vector potential on the vacuum side $z>0$, has recently been extended (Babiker and Barton 1976) to include the photon modes whose wavenumber q ranges from the plasma frequency, ω_p, to infinity. The vector potential, defined for $z>0$, is written as the sum of three separate parts

$$\boldsymbol{A}=\boldsymbol{A}_s+\boldsymbol{A}_p+\boldsymbol{A}_{sp}, \tag{4.1}$$

where the subscripts s and p refer to the two types of photon modes and sp refers to the surface plasmon modes. The s-photon part can be written as

$$\boldsymbol{A}_s=\sum_{\lambda=0,1,2}\int_{-\infty}^{\infty}\mathrm{d}^2\boldsymbol{k}\int_0^{\infty}\mathrm{d}q(a_{s\lambda}(\boldsymbol{k},q)f_\lambda(q,z)\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}.\boldsymbol{\rho}}+\mathrm{HC}, \tag{4.2}$$

where HC is the Hermitean conjugate and

$$f_\lambda(q,z)=\frac{\mathrm{i}\hat{k}\times\hat{z}}{\pi\omega_p}\left(\frac{q^2}{\omega}\right)^{1/2}\left\{\theta(\omega_p-q)\delta_{\lambda 0}\left(\cos qz+\frac{\nu}{q}\sin qz\right)\right.$$
$$\left.+\theta(q-\omega_p)\left(1-\frac{\Omega}{q}\right)^{1/2}\left[\delta_{\lambda 1}\cos qz-\delta_{\lambda 2}\left(\frac{\Omega}{q}\right)^{1/2}\sin qz\right]\right\}. \tag{4.3}$$

In the above expression θ is the step function and the variables ν and Ω are defined by

$$\nu^2=-\Omega^2=\omega_p^2-q^2,$$
$$\omega_p^2=\nu^2+q^2=q^2-\Omega^2. \tag{4.4}$$

For the p-polarized photons one has

$$\boldsymbol{A}_p=\sum_{\lambda=0,1,2}\int_{-\infty}^{\infty}\mathrm{d}^2\boldsymbol{k}\int_0^{\infty}\mathrm{d}q(a_{p\lambda}(\boldsymbol{k},q)\boldsymbol{g}_\lambda(q,z)\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}.\boldsymbol{\rho}}+\mathrm{HC}), \tag{4.5}$$

where

$$\boldsymbol{g}_\lambda(q,z)=\frac{\mathrm{i}}{\pi}\left(\frac{q^2}{\omega^3}\right)^{1/2}\left\{\frac{\theta(\omega_p-q)\delta_{\lambda 0}}{(1+\epsilon^2q^2/\nu^2)^{1/2}}\left[\left(\hat{k}\cos qz-\mathrm{i}\hat{z}\frac{k}{q}\sin qz\right)\right.\right.$$
$$\left.+\frac{\epsilon q}{\nu}\left(\hat{k}\sin qz+\mathrm{i}\hat{z}\frac{k}{q}\cos qz\right)\right]$$

$$+\frac{\theta(q-\omega_p)}{(1+\epsilon q/\Omega)^{1/2}}\left[\delta_{\lambda 1}\left(\hat{k}\cos qz - i\hat{z}\frac{k}{q}\sin qz\right)\right.$$
$$\left.\left.+\delta_{\lambda 2}\left(\frac{\epsilon q}{\Omega}\right)^{1/2}\left(\hat{k}\sin qz + i\hat{z}\frac{k}{q}\cos qz\right)\right]\right\} \tag{4.6}$$

with $\epsilon = 1-\omega_p^2/\omega^2$.

In equations (4.3) and (4.6) the functions preceded by $\theta(\omega_p - q)$, correspond to the vector potential used in previous calculations (Philpott 1975). The additional terms, preceded by $\theta(q-\omega_p)$, represent the photon modes transmitted into the plasma.

The surface plasmon part is

$$\boldsymbol{A}_{sp} = \int_{-\infty}^{\infty} d^2\boldsymbol{k}\left(\frac{1}{\pi p_k}\right)^{1/2}\left[a_{\boldsymbol{k}}\left(i\hat{k}+\hat{z}\frac{k}{q_0}\right)e^{i\boldsymbol{k}.\boldsymbol{\rho}-q_0 z}+\text{HC}\right], \tag{4.7}$$

where

$$q_0^2 = k^2 - \omega_k^2, \tag{4.8}$$

$$p_k = \frac{\epsilon_k^4 - 1}{\epsilon_k^2[-(1+\epsilon)]^{1/2}}, \qquad \epsilon_k = 1-\omega_p^2/\omega_k^2. \tag{4.9}$$

ω_k is the surface plasmon eigenfrequency corresponding to a surface plasmon with wavenumber k. The well known expression

$$k^2 = \omega_k^2\epsilon_k/(1+\epsilon_k), \tag{4.10}$$

relates k to ω_k. The annihilation operators $a_{s\lambda}(\boldsymbol{k}, q)$, $a_{p\lambda}(\boldsymbol{k}, q)$ and $a_{\boldsymbol{k}}$ obey the commutation rules

$$[a_{r\lambda}(\boldsymbol{k}, q), a^\dagger_{r'\lambda'}(\boldsymbol{k}', q')] = \delta_{rr'}\delta_{\lambda\lambda'}\delta(\boldsymbol{k}-\boldsymbol{k}')\delta(q-q'),$$
$$[a_{\boldsymbol{k}}, a^\dagger_{\boldsymbol{k}'}] = \delta(\boldsymbol{k}-\boldsymbol{k}'). \tag{4.11}$$

We shall treat the two-atom transition problem in front of the plasma surface only for configuration (a), which is shown in figure 1. The calculations for any other configuration can be conducted along similar lines.

The decay rates for the two-atom system in configuration (a) are given by

$$\Gamma^{(\pm)} = 2\pi\sum_Q |\langle\psi_\pm; 0|H_{\text{int}}|\psi_0; Q\rangle|^2\delta(\omega_0-\omega(Q)) \tag{4.12}$$

where the sum over the quanta, designated by Σ_Q is carried over all types of quanta. Q can be the quantum for an s photon, a p photon or a surface plasmon. $\omega(Q)$ is the energy of the quantum.

The decay rates are given by the sum of rates into the three orthogonal channels:

$$\Gamma^{(\pm)} = \Gamma_s^{(\pm)} + \Gamma_p^{(\pm)} + \Gamma_{sp}^{(\pm)}. \tag{4.13}$$

For the present configuration we have

$$\Gamma_s^{(\pm)} = 0. \tag{4.14}$$

We calculate in turn the contributions from the p photons and the surface plasmons.

The calculation of $\Gamma_p^{(\pm)}$ is typical of all calculations for decay rates into photon channels, for atomic systems in front of the plasma surface. The contributions from photons with $q<\omega_p$ and $q>\omega_p$ are first obtained separately. It will subsequently be observed that the two contributions can be combined, by means of a simple mathematical trick, so that the range of the q integration then extends from zero to infinity. The resulting expressions can then be dealt with by the methods of the previous sections.

We denote by $\Gamma_{p<}^{(\pm)}$ and $\Gamma_{p>}^{(\pm)}$ the contributions from the p photons with wavenumbers $q<\omega_p$ and $q>\omega_p$, respectively. Thus using equation (4.6) we obtain for $\Gamma_{p<}^{(\pm)}$,

$$\Gamma_{p<}^{(\pm)}=d^2\frac{(2\pi)^2}{2\pi^2}\int_0^\infty k\,dk\int_0^{\omega_p}dq\left[\frac{k^2}{\omega(1+\epsilon^2q^2/\nu^2)}S^{(\pm)}(kR)\right.$$
$$\left.\times\left(\sin^2 qz+\frac{\epsilon^2q^2}{\nu^2}\cos^2 qz+\frac{\epsilon q}{\nu}\sin 2qz\right)\delta(\omega_0-\omega)\right], \tag{4.15}$$

where $S^{(\pm)}(kR)$ is given by equation (3.7). Elementary trigonometry yields from equation (4.15)

$$\Gamma_{p<}^{(\pm)}=2d^2\int_0^\infty k\,dk\int_0^{\omega_p}dq\left\{\frac{k^2}{\omega}S^{(\pm)}(kR)\delta(\omega_0-\omega)\right.$$
$$\left.\times\left[1-\frac{1}{1+\epsilon^2q^2/\nu^2}\left(1-\frac{\epsilon^2q^2}{\nu^2}\cos 2qz-\frac{\epsilon q}{\nu}\sin 2qz\right)\right]\right\}. \tag{4.16}$$

The transmitted photon part, for which $q>\omega_p$, similarly gives the contribution

$$\Gamma_{p>}^{(\pm)}=2d^2\int_0^\infty k\,dk\int_{\omega_p}^\infty dq\left[\frac{k^2}{\omega}S^{(\pm)}(kR)\delta(\omega_0-\omega)\left(1+\frac{\epsilon q-\Omega}{\epsilon q+\Omega}\cos 2qz\right)\right]. \tag{4.17}$$

Equations (4.16) and (4.17) separate into z-independent and z-dependent parts. Thus we can write

$$\Gamma_p^{(\pm)}=\Gamma_{pa}^{(\pm)}+\Gamma_{pb}^{(\pm)}.$$

The z-independent parts are given by

$$\Gamma_{pa}^{(\pm)}=2d^2\int_0^\infty k\,dk\int_0^\infty dq\frac{k^2}{\omega}S^{(\pm)}(kR)\delta(\omega_0-\omega), \tag{4.18}$$

which, when expressed in terms of the variables ω and μ yields

$$\Gamma_{pa}^{(\pm)}=2d^2\omega_0^3\int_0^1 d\mu(1-\mu^2)S^{(\pm)}(\omega_0,\mu,R). \tag{4.19}$$

This is identical to equation (3.13). We have thus identified the decay rates for the isolated two-atom system.

The z-dependent parts of equations (4.16) and (4.17) combine to give $\Gamma_{pb}^{(\pm)}$ as follows:

$$\Gamma_{pb}^{(\pm)}=2d^2\int_0^\infty k\,dk\,\mathrm{Re}\int_0^\infty dq\left(e^{2iqz}\frac{k^2}{\omega}S^{\pm}(kR)\frac{\epsilon q-\Omega}{\epsilon q+\Omega}\delta(\omega_0-\omega)\right), \tag{4.20}$$

where Re denotes the 'real part'. When expressed in terms of the variables ω and μ, equation (4.20) becomes

$$\Gamma_{pb}^{(\pm)}(R,z)=2d^2\int_0^\infty \mathrm{d}\omega \mathrm{Re}\int_0^1 \mathrm{d}\mu\Big\{ e^{2i\omega\mu z}\omega^3(1-\mu^2)S^{(\pm)}(\mu,\omega,R)$$
$$\times\left(1-\frac{2\Omega(\mu,\omega)}{\mu\omega\epsilon+\Omega(\mu,\omega)}\right)\delta(\omega_0-\omega)\Big\}, \quad (4.21)$$

which finally yields

$$\Gamma_{pb}^{(\pm)}=2d^2\omega_0^3\mathrm{Re}\int_0^1 \mathrm{d}\mu\ e^{2i\omega_0\mu z}(1-\mu^2)S^{(\pm)}(\mu,\omega_0,R)$$
$$-4d^2\omega_0^3\mathrm{Re}\int_0^1 \mathrm{d}\mu\ e^{2i\omega_0\mu z}(1-\mu^2)\frac{\Omega(\mu,\omega_0)}{\mu\omega_0\epsilon+\Omega(\mu,\omega_0)}S^{(\pm)}(\mu,\omega_0,R). \quad (4.22)$$

The first term of equation (4.22) immediately gives the perfect conductor result of equation (3.14). The second part is a correction to the decay rates due to the finiteness of the plasma frequency.

Finally we consider the surface plasmon contributions. These are given by

$$\Gamma_{sp}^{(\pm)}=2\pi\int_{-\infty}^{\infty}\mathrm{d}^2\boldsymbol{k}|\langle\psi_\pm;0|H_{\mathrm{int}}|\psi_0;\boldsymbol{k}\rangle|^2\delta(\omega_0-\omega_k). \quad (4.23)$$

Using equations (4.7) we obtain from the above expression

$$\Gamma_{sp}^{(\pm)}=4\pi d^2\int_0^\infty k\ \mathrm{d}k\frac{1}{p_k}\frac{k^2}{q_0^2}S^{(\pm)}(kR)\,e^{-2q_0z}\delta(\omega_0-\omega_k). \quad (4.24)$$

The ω_k variables can be converted to k variables in the Dirac delta function by means of the relation

$$\delta(\omega_0-\omega_k)=\delta(k_0-k)\left|\frac{\mathrm{d}\omega_k}{\mathrm{d}k}\right|^{-1}_{k=k_0}=\frac{\omega_k}{k}\frac{\epsilon^2+1}{(1+\epsilon)^2}\delta(k_0-k). \quad (4.25)$$

The final result is (putting $\epsilon_0=\epsilon(\omega_0)$),

$$\Gamma_{sp}^{(\pm)}=\frac{3\pi\gamma_0\epsilon_0^3}{(\epsilon_0-1)[-(1+\epsilon_0)]^{5/2}}\{\exp[-2\omega_0z(-(1+\epsilon_0))^{-1/2}]\}\left(1\pm J_0\left(\left(\frac{\omega_0^2\epsilon_0}{1+\epsilon_0}\right)^{1/2}R\right)\right). \quad (4.26)$$

These define the decay rates into the surface plasmon channel. In the 'small-R' limit, we obtain

$$\Gamma_{sp}^{(\pm)}\doteq\frac{3\pi\gamma_0\epsilon_0^3}{(\epsilon_0-1)[-(1+\epsilon_0)]^{5/2}}\exp\{-2\omega_0z[-(1+\epsilon_0)]^{-1/2}\}(1\pm1+O(\omega_0^2R^2)). \quad (4.27)$$

These results show that the decay rate of the symmetric (+) state into the surface plasmon channel for the two-atom system satisfying $\omega_0R\ll1$ is twice the decay rate into the same channel for a single atom (Philpott 1975). The decay rate into this channel of the antisymmetric (−) state is negligible under these conditions.

4. Comments and conclusions

Specializing in some dipole orientations, we have obtained full expressions for the decay rates of the two-atom system in the presence of metallic surfaces. The latter are described as either conducting, and therefore impermeable to electromagnetic fields; or approximately represented by an electron plasma model which illustrates the effects of field penetration across the metallic surface.

Our results indicate that the transition rates are markedly modified close to the metallic surface. An excited atom in the vicinity of a conducting surface can have a lifetime four times shorter than the normal lifetime, when a similar atom in its ground state is placed close to it near the surface. There are other important contributions due to the surface plasmons, which are also enhanced in the presence of the resonating atom. The surface plasmon contributions are exponential functions of the distance from the surface and are consequently significant only close to the surface.

The so called monolayer assembly technique developed recently by Kuhn and co-workers (Kuhn *et al* 1972) provides the possibility of experimentally detecting the physical effects on atomic properties of the type mentioned in this paper, when systems of atoms are held close to material surfaces. Experiments along these have already been reported (see, for instance, Drexhage 1970, 1969).

Another aspect of the effects on the two-atom system is a modification of the resonance energy transfer between the two atoms, due to the presence of the surface. This has been briefly commented on by Philpott (1975). A full calculation, for both the cases of a perfect conductor and the plasma, may be conducted along the lines of Mclone and Power (1964) using the appropriate quantized vector potentials.

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